Influence of gauge artifact on adiabatic and entropy perturbations during inflation

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In recent publications we proposed one way to calculate gauge-invariant variables in the local observable universe, which is limited to a portion of the whole universe. To provide a theoretical prediction of the observable fluctuations, we need to preserve the gauge-invariance in the local universe, which is referred to the genuine gauge-invariance. The importance of the genuine gauge-invariance is highlighted in the study of primordial fluctuations in the infrared (IR) limit. The stability against IR loop corrections to primordial fluctuations is guaranteed in requesting the genuine gauge-invariance. The genuine gauge-invariance also gives the impact on the detectable fluctuations. We showed that at observable scales the bi-spectrum calculated in the conventional perturbation theory vanishes in the squeezed limit, if we request the genuine gauge invariance. This indicates that the conventional bi-spectrum is, in this limit, dominated by a gauge artifact, which cannot be observed. These studies have been elaborated in single-field models of inflation. In this paper we generalize our argument to multi-field models of inflation, where, in addition to the adiabatic field, the entropy field can participate in the generation of primordial fluctuations. We will find that the entropy field can generate the observable fluctuations, which cannot be eliminated by gauge transformations in the local universe.

I. INTRODUCTION

The conventional cosmological perturbation theory has been established based on the assumption that we know the whole spatial region of the universe with infinite volume. We should, however, recognize that this does not meet the case in the actual observations, because the observable portion of the universe is limited. In our recent works [1–4], we pointed out the necessity of distinguishing the gauge invariance in the whole universe with infinite volume from that in the local universe with finite volume. To preserve the gauge-invariance in the whole universe, it is sufficient to request the invariance under normalizable gauge transformations, which become regular at spatial infinity. However, in the local universe, where we need not to concern about the regularity at infinity, it is necessary to request the invariance under both normalizable and non-normalizable gauge transformations. We discriminate gauge-invariant quantities that can be constructed in our local observable universe, referring them as *genuine gauge-invariant variables*. The observable fluctuations should be such a genuinely gauge-invariant variable.

The genuine gauge-invariant perturbation has a large impact namely on the infrared (IR) modes of fluctuations. The adiabatic vacuum, which yields the scale-invariant spectrum, is supposed to be a natural vacuum in the inflationary universe. However, once the interaction turns on, it is not manifest whether the adiabatic vacuum is stable or unstable against the IR contributions in loop corrections [5–26]. (See also the recent discussions in Refs. [27–35].) In our previous works [2, 3], we showed that in single field models of inflation the IR divergence is an unphysical artifact, which disappears in requesting the genuine gauge-invariance. Namely, initial quantum states are requested to satisfy several conditions in order to respect the gauge invariance in the local universe. The importance of the genuine gauge invariance is recognized as well in the study of the primordial non-Gaussianity [4]. It is remarkable that the tree-level bi-spectrum calculated in the conventional perturbation theory vanishes in the squeezed limit under the request of the genuine gauge invariance. This fact emphasizes the importance to investigate primordial fluctuations based on the genuine gauge invariant perturbation theory to yield the theoretical prediction of fluctuations, which are to be compared with the observations.

These arguments have been so far elaborated in single field models of inflation. In this paper, we extend our argument to multi-field models. In multi-field models, the issue of gauge-invariance becomes more delicate because of the presence of the entropy field. If the entropy field is massless, the loop correction of the entropy field diverges due to IR contributions. In contrast to the IR divergence from the adiabatic field, the IR divergence from the entropy field is conceived to be irrelevant to the presence of the gauge degrees of freedom [36]. The purpose of this paper is not to provide ways of regularization, but to provide one way to realize the genuine gauge invariance in the presence of the entropy field. For this purpose, we need to discriminate between the IR divergences from the gauge effects and those from different origins. In this paper we consider two-field models of inflation, but an extension to inflation models with more than two fields would proceed straightforwardly.

Our paper is organized as follows. In Sec. II, we give the setup of our problem and after that we briefly review a method to calculate genuinely gauge-invariant quantities. In Sec. III, we study the implications of the genuine gauge-invariant perturbation in the two-field models with the pure adiabatic and entropy fields. After we derive the gauge-invariance condition, we calculate the genuinely gauge-invariant bi-spectrum in this model. In Sec. IV, we extend our arguments to more general two-field models where the adiabatic and entropy fields are coupled even at liner order. Our results are sum-

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marized in Sec. V.

II. GENUINE GAUGE-INVARIANT PERTURBATIONS

In this section, we first give the set up of the problem. After that, we provide one way to perform the genuinely gauge-invariant perturbation theory, following the discussion in our previous works [2, 3].

A. Basic equations

We consider the two-field inflation models with the standard kinetic term whose action take the form

$$S = \frac{M_{\rm pl}^2}{2}\int\sqrt{-g}\left[R - g^{\mu\nu}\phi^I_{,\mu}\phi_{I,\nu} - 2V(\phi_1,\,\phi_2)\right]{\rm d}^4x, \tag{2.1}$$

where $M_{\rm pl}$ is the Planck mass and multiplying $1/M_{\rm pl}$ the scalar fields were rescaled to be dimensionless. The field-space metric for ϕ^I is assumed to be given by the 2×2 unit matrix.

The ADM formalism has been utilized to derive the action of the dynamical variables particularly in the non-linear perturbation theory [43]. Using the decomposed metric

$${\rm d}s^2 = -N^2{\rm d}t^2 + h_{ij}({\rm d}x^i + N^i{\rm d}t)({\rm d}x^j + N^j{\rm d}t)\ , \tag{2.2}$$

the action is rewritten as

$$\begin{aligned} S \;=\; \frac{M_{\rm pl}^2}{2}\int\sqrt{h}\Big[&N\,{}^s\!R - 2NV(\phi_1,\,\phi_2) + \frac{1}{N}(E_{ij}E^{ij} - E^2) \\ &+\frac{1}{N}(\partial_t\phi_I - N^i\partial_i\phi_I)(\partial_t\phi^I - N^i\partial_i\phi^I) \\ &-Nh^{ij}\partial_i\phi_I\partial_j\phi^I\Big]{\rm d}^4x\ , \end{aligned} \tag{2.3}$$

where ${}^s\!R$ is the three-dimensional scalar curvature and E_{ij} and E are defined by

$$E_{ij} = \frac{1}{2}\left(\partial_t h_{ij} - D_iN_j - D_jN_i\right), \quad E = h^{ij}E_{ij} \tag{2.4}$$

with the three-dimensional covariant derivative D_i defined by h_{ij}. The spatial indices are raised and lowered by h_{ij}.

Perturbing the scalar fields as $\phi_I + \delta\phi_I$, the background equations of motion are derived as

$$\ddot{\phi}_I + 3\dot{\rho}\dot{\phi}_I + \partial V/\partial\phi^I = 0\,, \tag{2.5}$$

$$6\dot{\rho}^2 = \dot{\phi}_I\dot{\phi}^I + 2V(\phi_1,\phi_2)\,, \tag{2.6}$$

where the dot denotes the derivative by the cosmological time. Using these equations, we also have

$$\ddot{\rho} = -\frac{1}{2}\dot{\phi}_I\dot{\phi}^I\ . \tag{2.7}$$

B. Gauge-invariant operator

In this subsection, we describe our way to calculate the gauge-invariant quantity in the local universe. One simple way to preserve the gauge-invariance is to calculate fluctuations in a completely fixed slicing and threading. The time-slicing can be easily fixed by adapting the gauge condition at each spacetime point. By contrast, the complete fixing of the spatial coordinates is not easy-going in the local universe because of the degrees of freedoms in non-normalizable transformations [2, 3]. We therefore seek for the way to construct the manifestly invariant quantity under the change in the spatial coordinates instead of fixing them completely.

We fix the time slicing by eliminating the fluctuation in the adiabatic field. We also adapt the gauge conditions on the spatial coordinates, taking the spatial metric as

$$h_{ij} = e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}\ , \tag{2.8}$$

where $\delta\gamma_{ij}$ satisfies the transverse and traceless conditions $\delta\gamma^i{}_i = \partial_i\delta\gamma^i{}_j = 0$. While these conditions do not fix the spatial coordinates completely in the local universe, we need not worry about the presence of the residual gauge modes only if we consider the genuinely gauge-invariant quantity. We can construct the genuinely gauge-invariant operator by making use of the scalar quantities under the three dimensional spatial diffeomorphism. Following our previous works [2, 3], we pick up the scalar curvature ${}^s\!R$ as such a scalar quantity. Although scalar quantities also vary under the gauge transformation because of the change of their arguments x^i, this gauge ambiguity does not appear in the n-point functions of these quantities with the arguments specified in a gauge-invariant manner. In order to specify the arguments of the n-point functions in a gauge invariant manner, we measure distances from an arbitrary reference point to the n vertices by means of the geodesic distance obtained by solving the spatial three-dimensional geodesic equation:

$$\frac{{\rm d}^2x^i}{{\rm d}\lambda^2} + {}^s\Gamma^i{}_{jk}\frac{{\rm d}x^j}{{\rm d}\lambda}\frac{{\rm d}x^k}{{\rm d}\lambda} = 0\,, \tag{2.9}$$

where ${}^s\Gamma^i{}_{jk}$ is the Christoffel symbol with respect to the three dimensional spatial metric on a constant time hypersurface and λ is the affine parameter. We consider the three-dimensional geodesics whose affine parameter ranges from $\lambda = 0$ to 1 with the initial "velocity" given by

$$\left.\frac{{\rm d}x^i(\boldsymbol{X},\lambda)}{{\rm d}\lambda}\right|_{\lambda=0} = e^{-\zeta(\lambda=0)}\left[e^{-\delta\gamma(\lambda=0)/2}\right]^i{}_jX^j\ . \tag{2.10}$$

We identify a point in the geodesic normal coordinates X^i with the end point of the geodesic, $x^i(\boldsymbol{X},\lambda=1)$. Expanding the global coordinates x^i by the geodesic normal coordinates X^i as $x^i(\boldsymbol{X}) =: X^i + \delta x^i(\boldsymbol{X})$, a genuine gauge invariant variable can be given by

$$\begin{aligned} {}^g\!R(X) &:= {}^s\!R(t,\,x^i(\boldsymbol{X})) \\ &= \sum_{n=0}^{\infty}\frac{\delta x^{i_1}\cdots\delta x^{i_n}}{n!}\partial_{X^{i_1}}\cdots\partial_{X^{i_n}}{}^s\!R(t,\,X^i), \end{aligned} \tag{2.11}$$

where we introduced the abbreviated notation $X := \{t,\, \boldsymbol{X}\}$. Similar quantities to our genuine gauge-invariant variable were studied in Refs. [37–41], while the authors of Refs. [37–40] seem to have the different opinions regarding what are actual observables.

In the following section, we use the equality $\stackrel{\rm IR}{\approx}$ which is valid only when we neglect the terms which do not yield IR divergences in the one-loop corrections to $\langle {}^g\!R {}^g\!R \rangle$. When we write down the Heisenberg operator in terms of the interaction picture fields for the adiabatic and entropy fields, this is equivalent to keep only terms without spatial/time derivatives and the terms which include only one interaction picture field with differentiation. In deriving the gauge-invariance condition from the IR regularity of $\langle {}^g\!R {}^g\!R \rangle$, we neglect the possibly divergent loop corrections whose loop integrals are composed only of the entropy field, because they are irrelevant to the gauge effects. Using the equality $\stackrel{\rm IR}{\approx}$, the geodesic normal coordinates X^i are related to the global coordinates x^i as

$$x^i(\boldsymbol{X}) \stackrel{\rm IR}{\approx} e^{-\zeta} \left[e^{-\delta\gamma/2} \right]^i{}_j X^j\,. \tag{2.12}$$

Abbreviating the unimportant prefactor, we simply denote the scalar curvature ${}^s\!R$ as

$${}^s\!R \stackrel{\rm IR}{\approx} e^{-2\zeta} \left[e^{-\delta\gamma} \right]^{ij} \partial_{ij}\zeta\,. \tag{2.13}$$

The gravitational wave field $\delta\gamma_{ij}$ can also generate the IR divergent with $\langle \delta\gamma_{ij}\delta\gamma_{kl} \rangle$. We can easily confirm that these contributions are generated only through the interaction between the adiabatic field and the gravitational wave field and that the entropy field does not produce another potentially divergent terms with the gravitational wave field. Therefore, repeating the same argument as in Ref. [3], we can show that the contributions from the gravitational waves are canceled in the gauge-invariant operator ${}^g\!R$. To avoid the repetition of calculations, we here do not explicitly write down the contributions from the gravitational wave field.

Expanding ζ as $\zeta = \zeta_1 + \zeta_2 + \zeta_3 + ...$ where we denote ζ_1 as $\psi := \zeta_1$, the spatial curvature ${}^s\!R$ is expressed as

$$\begin{aligned} &{}^s\!R_1 = \partial^2\psi\,, \qquad {}^s\!R_2 \stackrel{\rm IR}{\approx} \partial^2\zeta_2 - 2\psi\partial^2\psi\,, \\ &{}^s\!R_3 \stackrel{\rm IR}{\approx} \partial^2\zeta_3 - 2(\zeta_2\partial^2\psi + \psi\partial^2\zeta_2) + 2\psi^2\partial^2\psi\,. \end{aligned} \tag{2.14}$$

Using Eq. (2.12), the difference between the geodesic normal coordinates and the global ones is expanded as

$$\delta x^i = \delta x^i_1 + \delta x^i_2 + \cdots \tag{2.15}$$

where

$$\delta x^i_1 \stackrel{\rm IR}{\approx} -\psi X^i, \qquad \delta x^i_2 \stackrel{\rm IR}{\approx} -\zeta_2 X^i + \frac{1}{2}\psi^2 X^i\,. \tag{2.16}$$

Using Eqs. (2.11), (2.14), and (2.16), we have

$${}^g\!R_1 = \partial^2\psi\,, \tag{2.17}$$

$${}^g\!R_2 \stackrel{\rm IR}{\approx} \partial^2\zeta_2 - \psi\partial^2 X^i\partial_{X^i}\psi\,, \tag{2.18}$$

$$\begin{aligned} {}^g\!R_3 \stackrel{\rm IR}{\approx}\ & \partial^2\zeta_3 - \zeta_2\partial^2(X^i\partial_{X^i})\psi \\ & - \psi\partial^2(X^i\partial_{X^i})\zeta_2 + \frac{1}{2}\psi^2\partial^2(X^i\partial_{X^i})^2\psi\,. \end{aligned} \tag{2.19}$$

III. MODELS WITH THE PURE ENTROPY FIELD

In this and next sections, we investigate the predictions of the genuine gauge-invariant perturbation theory in two kinds of models. In general, the adiabatic and entropy fields can be coupled even in liner order. We first study a particular model where these two-fields are decoupled at linear order, deferring the study of coupled models to the next section.

A. Non-linear perturbations

In this section, we consider the two-field model where the field ϕ_1 dominates the background. We use the horizon flow functions [42] defined in a similar way to those in single field models as

$$\varepsilon_1 := \frac{\dot\phi_1^2}{2\dot\rho^2}, \qquad \varepsilon_{m+1} := \frac{1}{\varepsilon_m}\frac{{\rm d}\varepsilon_m}{{\rm d}\rho} \qquad \text{for } m \geq 1. \tag{3.1}$$

Assuming that the horizon flow functions ε_m are all small of $\mathcal{O}(\varepsilon)$, we neglect the terms of $\mathcal{O}(\varepsilon^3)$. We also employ the assumption:

$$\frac{\dot\phi_2}{\dot\phi_1} = \mathcal{O}(\varepsilon^2)\,. \tag{3.2}$$

This condition requests that, within our approximation, the field ϕ_1 and ϕ_2 become the pure adiabatic and entropy fields, respectively. We define the gauge-invariant quantity ${}^g\!R$ on the slicing:

$$\delta\phi_1 = 0\,. \tag{3.3}$$

To evaluate the genuine gauge-invariant variable, it is convenient to perform the calculation in the flat gauge:

$$\tilde h_{ij} = e^{2\rho}\left[e^{\delta\tilde\gamma} \right]_{ij}, \quad \delta\tilde\gamma^i{}_i = 0 = \partial_i\delta\tilde\gamma^i_j, \tag{3.4}$$

where all the interaction vertexes are explicitly suppressed by the slow-roll parameters [1, 43]. We associate a tilde with the metric perturbations in the flat gauge to discriminate those in the comoving gauge. Here again we can neglect the contributions from the gravitational wave field, which do not appear in

${}^{g}\!R$. The action in this gauge is given by

$$S \overset{\rm IR}{\approx} \frac{M_{\rm pl}^2}{2}\int {\rm d}t\,{\rm d}^3\boldsymbol{x} e^{3\rho}\Big[\tilde{N}^{-1}(-6\dot{\rho}^2+4\dot{\rho}\partial_i\tilde{N}^i) + \tilde{N}^{-1}\Big(\dot{\phi}_I+\dot{\varphi}_I-\tilde{N}^i\partial_i\varphi_I\Big)\Big(\dot{\phi}^I+\dot{\varphi}^I-\tilde{N}^i\partial_i\varphi^I\Big) - 2\tilde{N}\sum_{m=0}^{\infty}\frac{V_{I_1\cdots I_m}}{m!}\varphi^{I_1}\cdots\varphi^{I_m} - \tilde{N}\tilde{h}^{ij}\partial_i\varphi_I\partial_j\varphi^I\Big], \tag{3.5}$$

where we introduced $V_{I_1\cdots I_n} := \partial^n V/\partial\phi^{I_1}\cdots\partial\phi^{I_n}$ and $\delta\phi_I := \varphi_I$. Using the background equations and Eq. (3.2), the derivatives of the potential are quantified as

$$\frac{V_2}{\dot{\rho}\dot{\phi}_1} = \mathcal{O}(\varepsilon^2)\,, \quad \frac{V_{12}}{\dot{\rho}^2} = \mathcal{O}(\varepsilon^3)\,, \quad \frac{V_{112}}{\dot{\rho}\dot{\phi}_1} = \mathcal{O}(\varepsilon^2)\,. \tag{3.6}$$

The curvature perturbation in the gauge $\delta\phi_1 = 0$ is related to the fluctuation of the dimensionless scalar field (divided by $M_{\rm pl}$) in the flat gauge φ_1 as

$$\zeta \overset{\rm IR}{\approx} \zeta_n + \zeta_n\partial_\rho\zeta_n + \frac{\varepsilon_2}{4}\zeta_n^2 + \frac{\zeta_n^2\partial_\rho^2\zeta_n}{2} + \frac{3\varepsilon_2\zeta_n^2\partial_\rho\zeta_n}{4} + \frac{1}{12}\varepsilon_2(\varepsilon_2+2\varepsilon_3)\zeta_n^3, \tag{3.7}$$

where we have introduced

$$\zeta_n := -(\dot{\rho}/\dot{\phi}_1)\,\varphi_1\,. \tag{3.8}$$

Variation of the total action with respect to φ_I yields

$$e^{-3\rho}\partial_t\left[\frac{e^{3\rho}}{\tilde{N}}\left(\dot{\phi}_I+\dot{\varphi}_I\right)\right] + \tilde{N}\sum_{m=0}\frac{V_{II_1\cdots I_m}}{m!}\varphi^{I_1}\cdots\varphi^{I_m} - \left(\dot{\phi}_I+\dot{\varphi}_I\right)\frac{1}{\tilde{N}}\partial_i\tilde{N}^i - \tilde{N}e^{-2\rho}\partial^2\varphi_I \overset{\rm IR}{\approx} 0\,. \tag{3.9}$$

Variations with respect to the lapse function and the shift vector yield the Hamiltonian constraint:

$$(\tilde{N}^2-1)V + \tilde{N}^2\sum_{m=1}\frac{V_{I_1\cdots I_m}}{m!}\varphi^{I_1}\cdots\varphi^{I_m} + 2\dot{\rho}\partial_i\tilde{N}^i + \dot{\phi}_I\dot{\varphi}^I + \frac{1}{2}\dot{\varphi}_I\dot{\varphi}^I \overset{\rm IR}{\approx} 0\,, \tag{3.10}$$

and the momentum constraints:

$$2\dot{\rho}\partial_i\tilde{N} - \tilde{N}(\dot{\phi}_I\partial_i\varphi^I + \partial_i\varphi_I\dot{\varphi}^I) \overset{\rm IR}{\approx} 0\,. \tag{3.11}$$

From the form of the constraint equations, we can confirm the presence of the degrees of freedom which appear from boundary conditions in solving these equations. Repeating a similar analysis to the one in Ref. [2], these degrees of freedom are found to be the residual gauge degrees of freedom, which can change the average value of ζ_n at each order in perturbation.

These constraint equations are solved to give

$$\delta\tilde{N} \overset{\rm IR}{\approx} -\varepsilon_1\zeta_n + \frac{\varepsilon_1^2}{2}\zeta_n^2 + \frac{1}{4}\varepsilon_1\varepsilon_2\left(\zeta_n^2+\sigma_n^2\right), \tag{3.12}$$

$$\frac{1}{\dot{\rho}}\partial_i\tilde{N}^i \overset{\rm IR}{\approx} \varepsilon_1\partial_\rho\zeta_n - \frac{1}{2}\varepsilon_1\varepsilon_2\left(\zeta_n\partial_\rho\zeta_n + \sigma_n\partial_\rho\sigma_n\right) - \frac{1}{4}\varepsilon_1\left(2\eta_{22}+3\varepsilon_2\right)\sigma_n^2\,, \tag{3.13}$$

where, in a similar manner to ζ_n, we introduced

$$\sigma_n := -(\dot{\rho}/\dot{\phi}_1)\,\varphi_2\,. \tag{3.14}$$

Substituting Eqs. (3.12) and (3.13) into Eq. (3.9), the evolution equation of ζ_n is recast into a rather compact expression,

$$\mathcal{L}\zeta_n \overset{\rm IR}{\approx} \left[-2\varepsilon_1\zeta_n + \frac{1}{2}\varepsilon_1(4\varepsilon_1+\varepsilon_2)\zeta_n^2\right]\frac{e^{-2\rho}}{\dot{\rho}^2}\partial^2\zeta_n - \varepsilon_1\varepsilon_2\zeta_n\partial_\rho\zeta_n - \frac{3}{4}\varepsilon_2\varepsilon_3\zeta_n^2\,, \tag{3.15}$$

where the differential operator $\mathcal{L}$ is defined by

$$\mathcal{L} := \partial_\rho^2 + (3-\varepsilon_1+\varepsilon_2)\partial_\rho - \frac{e^{-2\rho}}{\dot{\rho}^2}\partial^2. \tag{3.16}$$

Note that the adiabatic field ζ_n is decoupled from the entropy field σ_n at linear order as the result of requiring Eq. (3.2). We also find that the interaction terms with $\zeta_n\sigma_n$ and $\zeta_n^2\sigma_n$ are suppressed at the order of $\mathcal{O}(\varepsilon^3)$. Since the entropy field s_n is not included in Eqs. (3.7) and (3.15), the curvature perturbation ζ and the gauge-invariant spatial curvature ${}^{g}\!R$ are given in the same form as those in single field models. Therefore, in the decoupled model which satisfies Eq. (3.2), the entropy field s_n does not contribute to the IR divergence that originates from the gauge effect.

Reflecting these facts, the gauge-invariance conditions derived by requesting the regularity of loop corrections from the adiabatic field take the same form as those in the single field model. We briefly describe this result, deferring the detailed explanation to Ref. [3]. Here, we expand the interaction picture field ψ as

$$\psi(X) = \int\frac{{\rm d}^3\boldsymbol{k}}{(2\pi)^{3/2}}\left[\,\psi_{\boldsymbol{k}}(X)a_{\boldsymbol{k}} + {\rm h.c.}\,\right] \tag{3.17}$$

with the creation and annihilation operators that satisfy

$$\left[a_{\boldsymbol{k}},\, a_{\boldsymbol{k}'}^\dagger\right] = \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,. \tag{3.18}$$

If the positive frequency function satisfy

$$\Big[\big\{1+\varepsilon_1(1+\varepsilon_1+\varepsilon_2)\big\}\partial_\rho - X^i\partial_{X^i} + \varepsilon_1 + \frac{1}{2}\varepsilon_2 + 2\xi_2 - \frac{3}{2}\mathcal{L}_k^{-1}\varepsilon_2(2\varepsilon_1+\varepsilon_2)\Big]\psi_{\boldsymbol{k}}(X) = -D_k\psi_{\boldsymbol{k}}(X)\,, \tag{3.19}$$

where D_k and $\mathcal{L}_k$ are defined as

$$D_k := \boldsymbol{k}\cdot\partial_{\boldsymbol{k}} + 3/2\,, \tag{3.20}$$

$$\mathcal{L}_k := \partial_\rho^2 + (3-\varepsilon_1+\varepsilon_2)\partial_\rho + \frac{e^{-2\rho}}{\dot{\rho}^2}k^2\,, \tag{3.21}$$

the gauge invariant operator ${}^{g}\!R_2$ is compactly summed as

$${}^{g}\!R_2 \overset{\rm IR}{\approx} -(1+\lambda_2)\partial^2 D_k\psi\,. \tag{3.22}$$

Here ξ_2 and λ_2 are time-dependent functions of $\mathcal{O}(\varepsilon^2)$ and they appeared as degrees of freedom in solving Eq. (3.15) at second order in perturbation. We can also find similar degrees of freedom at third order in perturbation and we label them by time dependent functions ξ_3 and λ_3 of $\mathcal{O}(\varepsilon^2)$. If we choose them correctly [3], the gauge invariant operator ${}^gR_3$ is compactly rewritten as

$$ {}^gR_3 \overset{\rm IR}{\approx} \frac{1}{2}\psi^2\partial^2\left[(1+2\lambda_2)D_k^2\psi - \mu D_k\psi\right] - \delta\zeta_{n,2}\partial^2 D_k\psi\,, \tag{3.23} $$

where we defined $\delta\zeta_{n,2}$ and μ as

$$ \delta\zeta_{n,2} := -\mathcal{L}^{-1}\frac{3}{4}\varepsilon_2(2\varepsilon_1+\varepsilon_2)\psi^2\,, \tag{3.24} $$

$$ \mu := \varepsilon_1 + \frac{1}{2}\varepsilon_2 - 3\varepsilon_1^2 + \frac{1}{2}\varepsilon_1\varepsilon_2 + 2(\xi_2+\lambda_3)\,. \tag{3.25} $$

We finally find that the possibly divergent terms become the total derivative form as

$$ \begin{aligned} &\langle\{{}^gR(X_1),\,{}^gR(X_2)\}\rangle \\ &\overset{\rm IR}{\approx} \frac{1}{2}\langle\psi^2\rangle\int\frac{\mathrm{d}(\log k)}{2\pi^2}\left\{(1+2\lambda_2)\partial^2_{\log k} - \mu\partial_{\log k}\right\} \\ &\qquad\qquad\times\left\{k^7\psi_{\boldsymbol{k}}(X_1)\psi^*_{\boldsymbol{k}}(X_2) + (\text{c.c.})\right\} \\ &\quad - \langle\delta\zeta_{n,2}\rangle\int\frac{\mathrm{d}(\log k)}{2\pi^2}\partial_{\log k}\left\{k^7\psi_{\boldsymbol{k}}(X_1)\psi^*_{\boldsymbol{k}}(X_2) + (\text{c.c.})\right\}\,, \end{aligned} \tag{3.26} $$

and hence they vanish. Here we symmetrized about X_1 and X_2. We thus obtain the gauge-invariance conditions by requesting the regularity of the loop corrections from the adiabatic field. Note that the preservation of the genuine gauge-invariance guarantees the regularity of the loop corrections from the adiabatic field, but it does not guarantee the regularity of those from the entropy field. Therefore, the divergent terms with $\langle\chi^2\rangle$ are still left in the left-hand side of Eq. (3.26) and they should be regularized by another method.

B. Primordial non-Gaussianity

In this subsection, we study the primordial non-Gaussianity, requesting the gauge-invariance conditions. In calculating the tree-level bi-spectrum, we can construct the genuine gauge-invariant variable using the curvature perturbation ζ in stead of the spatial scalar curvature ${}^sR$. If we consider the modes with $k \gg 1/L_{\rm obs}$, where $L_{\rm obs}$ denotes the observable scale in the comoving coordinates, the curvature perturbation ζ evaluated in the geodesic normal coordinates:

$$ \begin{aligned} {}^g\zeta(X) &:= \zeta(\rho,\,x^i(\boldsymbol{X})) \\ &= \zeta(X) + \delta x^i\partial_i\zeta|_{x^i=X^i} + \cdots \end{aligned} \tag{3.27} $$

becomes the gauge-invariant operator at least up to the second order in perturbation [4].

Introducing the Fourier modes of ${}^g\zeta$ as

$$ {}^g\zeta_{\boldsymbol{k}}(\rho) = \int\frac{\mathrm{d}^3\boldsymbol{X}}{(2\pi)^{3/2}}\,e^{-i\boldsymbol{k}\cdot\boldsymbol{X}}\,{}^g\zeta(\rho,\,\boldsymbol{X})\,, \tag{3.28} $$

we calculate the bispectrum of ${}^g\zeta$s at the leading order in perturbation. Expanding ${}^g\zeta$ as ${}^g\zeta_{\boldsymbol{k}} = \psi_{\boldsymbol{k}} + {}^g\zeta_{\boldsymbol{k},2} + \cdots$, we have

$$ \begin{aligned} &\langle{}^g\zeta_{\boldsymbol{k}_1}{}^g\zeta_{\boldsymbol{k}_2}{}^g\zeta_{\boldsymbol{k}_3}\rangle \\ &= \langle\psi_{\boldsymbol{k}_1}\psi_{\boldsymbol{k}_2}{}^g\zeta_{\boldsymbol{k}_3,2}\rangle + \langle\psi_{\boldsymbol{k}_1}{}^g\zeta_{\boldsymbol{k}_2,2}\psi_{\boldsymbol{k}_3}\rangle + \langle{}^g\zeta_{\boldsymbol{k}_1,2}\psi_{\boldsymbol{k}_2}\psi_{\boldsymbol{k}_3}\rangle\,. \end{aligned} \tag{3.29} $$

In the calculation of the bi-spectrum for ${}^g\zeta$, we again use the equality $\overset{\rm IR}{\approx}$, which picks up gauge-dependent terms. Using Eqs. (2.16) and (3.7), the gauge-invariant curvature perturbation is expressed as

$$ {}^g\zeta \overset{\rm IR}{\approx} \zeta_n + \zeta_n\left(\partial_\rho - X^i\partial_{X^i}\right)\zeta_n + \frac{1}{4}\varepsilon_2\zeta_n^2\,, \tag{3.30} $$

where ζ_n is given by solving Eq. (3.15). Since there is no contribution from the entropy field in Eqs. (3.15) and (3.30), ${}^g\zeta$ takes the same form as in single field models. As expected from these facts, repeating the same calculation as in Ref. [4], we again find that, in the squeezed limit with $k_1 \ll k_2 \approx k_3$, the bi-spectrum vanishes as

$$ \langle{}^g\zeta_{\boldsymbol{k}_1}{}^g\zeta_{\boldsymbol{k}_2}{}^g\zeta_{\boldsymbol{k}_3}\rangle \overset{\rm IR}{\approx} 0\,. \tag{3.31} $$

The bi-spectrum calculated in the conventional perturbation theory is thus found to be dominated by the gauge artifact also in the two-field model which satisfies the condition (3.2). Note that, while the three-point function generated from the cubic interaction ζ_n^3 is suppressed from the request of the genuine gauge invariance, the one generated from the interaction $\zeta_n\sigma_n^2$ is not suppressed at all. Therefore, our gauge-invariance conditions do not affect on the three-point function with one adiabatic field and two entropy fields.

One important remark is in order regarding the applicability of our argument. Precisely speaking, the expression of δx^i given in Eq. (2.16) can be reliably used only for the modes with $k \ll 1/L_{\rm obs}$. We can however show that the additional terms which appear for $k \gtrsim 1/L_{\rm obs}$ vanish separately in the squeezed limit [3]. Thus, we can confirm that the result given in Eq. (3.31) is robust as long as we consider the super-horizon modes with $k \ll e^\rho H$. During inflation, the scales relevant to the current observations go far outside of the inflationary horizon, which leads to the natural assumption that the observable scale is much larger than the horizon scale $1/H$. Therefore, our result is applicable to the observable modes which satisfy $1/L_{\rm obs} \ll k \ll e^\rho H$.

IV. GENERAL EXTENSION

In the previous section, we studied the two-field model of inflation where the adiabatic and entropy fields decouple at linear order in perturbation. In this model, the allowable form of interactions is significantly restricted so that the entropy

field does not contribute to the potentially divergent terms with the loop integrals of the adiabatic field. The predictions of the genuinely gauge-invariant perturbation are then found to be essentially same as those in single field models of inflation. In this section, we consider more general two-field models where the adiabatic and entropy fields are coupled even at linear order. The study of these models would help us to understand how the entropy field contributes to the IR corrections affected by non-local gauge degrees of freedom.

A. Adiabatic and entropy perturbations

In multi-field models, we need to care about the fact that the IR modes include the physical degrees of freedom as well as the gauge degrees of freedom. The discrimination of the later from the former proceeds easily in the model with the pure adiabatic and entropy fields, but in general it becomes more complicated. To ease this procedure, we decompose the fluctuations in the two fields into the horizontal and orthogonal directions to the background trajectory as

$$\begin{pmatrix} \delta\sigma \\ \delta s \end{pmatrix} = \frac{1}{\dot\sigma}\begin{pmatrix} \dot\phi_1 & \dot\phi_2 \\ -\dot\phi_2 & \dot\phi_1 \end{pmatrix}\begin{pmatrix} \delta\phi_1 \\ \delta\phi_2 \end{pmatrix} =: \Theta \begin{pmatrix} \delta\phi_1 \\ \delta\phi_2 \end{pmatrix}, \quad (4.1)$$

where we introduced the 2×2 time-dependent rotational matrix Θ with $\dot\sigma$ defined as

$$\dot\sigma := \sqrt{(\dot\phi_1)^2 + (\dot\phi_2)^2}\,.$$

In this expression, the action (2.3) is rewritten as

$$\begin{aligned} S = \frac{M_{\rm pl}^2}{2}\int {\rm d}^4x\,\sqrt{h}\Big[& N\,{}^sR - 2NV(\sigma,\, s) + \frac{1}{N}\left(E^{ij}E_{ij} - E^2\right) \\ & + \frac{1}{N}\left(\dot\sigma + \delta\dot\sigma - N^i\partial_i\delta\sigma - \dot\theta\delta s\right)^2 - Nh^{ij}\partial_i\delta\sigma\partial_j\delta\sigma \\ & + \frac{1}{N}\left(\delta\dot s - N^i\partial_i\delta s + \dot\theta\delta\sigma\right)^2 - Nh^{ij}\partial_i\delta s\partial_j\delta s\Big], \end{aligned} \quad (4.2)$$

where θ is the local rotation angle given by

$$\theta := \tan^{-1}(\dot\phi_2/\dot\phi_1)\,.$$

The model studied in the previous section corresponds to the particular case with $\theta =$const, where the background trajectory is not curved.

Using $\delta\sigma$ and δs, we rewrote the perturbation expansion of the potential as

$$\begin{aligned} V(\sigma,\, s) = {} & V_0 + V_\sigma\delta\sigma + V_s\delta s \\ & + \frac{1}{2}V_{\sigma\sigma}\delta\sigma^2 + V_{\sigma s}\delta\sigma\delta s + \frac{1}{2}V_{ss}\delta s^2 + \cdots\,, \end{aligned}$$

where V_0 denotes the background value of the potential. Note that $V_{\alpha_1..\alpha_i..\alpha_n}$ where $\alpha_i = \sigma,\, s$ are not the derivatives of the potential in terms of σ and s, but they are given by the linear combinations of $V_{I_1..I_n}$. In general multi-field models, the cross correlation between the adiabatic and entropy fields $\langle\delta\sigma\delta s\rangle$ does not necessarily vanish. The loop corrections with $\langle\delta\sigma\delta s\rangle$ can also yield divergent terms which are possibly affected by gauge degrees of freedom. Keeping this in mind, here we consider the terms that can contribute to the loop corrections with $\langle\delta\sigma^2\rangle$ and also $\langle\delta\sigma\delta s\rangle$. As in the previous section, we keep terms that include at most one interaction picture field for $\delta\sigma$ or δs with differentiation, but among them we neglect terms that are expanded only in terms of the interaction picture field for δs. In the following we use the same equality $\overset{\rm IR}{\approx}$ to denote a equality which is valid in neglecting these terms.

We define the gauge-invariant scalar curvature ${}^gR$ on the time-slice fixed by the gauge condition:

$$\delta\sigma = 0.$$

We again change the gauge into the flat gauge given by Eq. (3.4). Noticing the fact that the background evolution is characterized only by the adiabatic field, we introduce the horizon-flow functions as

$$\varepsilon_1 := \frac{\dot\sigma^2}{2\dot\rho^2}, \qquad \varepsilon_{m+1} := \frac{1}{\varepsilon_m}\frac{{\rm d}\varepsilon_m}{{\rm d}\rho} \qquad \text{for } m \geq 1. \quad (4.3)$$

In the following, we assume that the change in the background trajectory takes place satisfying ${\rm d}\theta/{\rm d}\rho = \mathcal{O}(\varepsilon)$. We also assume that the mass of the entropy field satisfies $V_{ss}/\dot\rho^2 = \mathcal{O}(\varepsilon)$ so that the entropy field participates in the generation of the fluctuation. In the flat slicing, the action is given by

$$\begin{aligned} S \overset{\rm IR}{\approx} \frac{M_{\rm pl}^2}{2}\int {\rm d}t\,{\rm d}^3\boldsymbol{x} e^{3\rho}\Big[& \tilde N^{-1}(\dot\sigma + \delta\dot\sigma - \tilde N^i\partial_i\delta\sigma - \dot\theta\delta s)^2 \\ & + \tilde N^{-1}(\delta\dot s - \tilde N^i\partial_i\delta s + \dot\theta\delta\sigma)^2 - \tilde N\tilde h^{ij}\partial_i\delta\sigma\partial_j\delta\sigma \\ & - \tilde N\tilde h^{ij}\partial_i\delta s\partial_j\delta s + \tilde N^{-1}(-6\dot\rho^2 + 4\dot\rho\partial_i\tilde N^i) - 2\tilde NV\Big]\,. \end{aligned} \quad (4.4)$$

The constraint equations are derived as

$$\begin{aligned} & 2\dot\rho\partial_i\tilde N^i + \dot\sigma\delta\dot\sigma + \frac{1}{2}(\delta\dot\sigma - \dot\theta\delta s)^2 + \frac{1}{2}(\delta\dot s + \dot\theta\delta\sigma)^2 \\ & \quad + (\tilde N^2 - 1)V_0 + V_\sigma\delta\sigma \\ & \quad + \frac{1}{2}(V_{\sigma\sigma}\delta\sigma^2 + 2V_{\sigma s}\delta\sigma\delta s + V_{ss}\delta s^2) + \cdots \overset{\rm IR}{\approx} 0\,, \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} & 2\dot\rho\partial_i\tilde N - \tilde N(\dot\sigma - \dot\theta\delta s)\partial_i\delta\sigma - \tilde N\dot\theta\delta\sigma\partial_i\delta s \\ & \qquad - 2\tilde N(\delta\dot\sigma\partial_i\delta\sigma + \delta\dot s\partial_i\delta s) \overset{\rm IR}{\approx} 0\,, \end{aligned} \quad (4.6)$$

where, in Eq. (4.5), we abbreviated the higher-order terms in $V(\sigma,\, s)$. In deriving the Hamiltonian constraint (4.5), we used $V_s = -\dot\sigma\dot\theta$, which is given by varying the action with respect to δs. Consulting Eqs. (4.5) and (4.6), we again find that the degrees of freedom in the boundary conditions are intrinsically attributed to the gauge degrees of freedom in the adiabatic fluctuation, while the entropy fluctuations are, at non-linear order, also affected by the gauge modes through the

change in the lower-order adiabatic fluctuation. Varying the action with respect to $\delta\sigma$, we have the equation of motion for $\delta\sigma$ as

$$e^{-3\rho}\partial_t\left\{\frac{e^{3\rho}}{\tilde N}(\dot\sigma+\delta\dot\sigma-\dot\theta\delta s)\right\}-(\dot\sigma+\delta\dot\sigma-\dot\theta\delta s)\frac{\partial_i\tilde N^i}{\tilde N}$$
$$+\tilde N\Big(V_\sigma+V_{\sigma\sigma}\delta\sigma+V_{\sigma s}\delta s$$
$$+\frac12 V_{\sigma\sigma\sigma}\delta\sigma^2+\frac12 V_{\sigma ss}\delta s^2+V_{\sigma\sigma s}\delta\sigma\delta s\Big)$$
$$-\frac{\dot\theta}{\tilde N}(\delta\dot s+\dot\theta\delta\sigma)-\tilde N e^{-2\rho}\partial^2\delta\sigma\overset{\rm IR}{\approx}0\,. \tag{4.7}$$

From a similar calculation to that in Appendix A of Ref. [3], the relation between the fluctuations in these two gauges is obtained as

$$\zeta\overset{\rm IR}{\approx}\zeta_n+\zeta_n\partial_\rho\zeta_n+\frac14\varepsilon_2\zeta_n^2-\frac{{\rm d}\theta}{{\rm d}\rho}\zeta_n s_n$$
$$+\frac12\zeta_n^2\partial_\rho^2\zeta_n+\frac34\varepsilon_2\zeta_n^2\partial_\rho\zeta_n+\frac1{12}\varepsilon_2(\varepsilon_2+2\varepsilon_3)\zeta_n^3$$
$$-\frac13\Big(\frac{{\rm d}\theta}{{\rm d}\rho}\Big)^2\zeta_n^3-2\frac{{\rm d}\theta}{{\rm d}\rho}\zeta_n s_n\partial_\rho\zeta_n-\frac{{\rm d}\theta}{{\rm d}\rho}\zeta_n^2\partial_\rho s_n$$
$$-\frac12\left(\frac{{\rm d}^2\theta}{{\rm d}\rho^2}+\frac32\varepsilon_2\frac{{\rm d}\theta}{{\rm d}\rho}\right)\zeta_n^2 s_n+\left(\frac{{\rm d}\theta}{{\rm d}\rho}\right)^2\zeta_n s_n^2\,. \tag{4.8}$$

where we defined ζ_n and s_n as

$$\zeta_n:=-(\dot\rho/\dot\sigma)\delta\sigma\,,\qquad s_n:=-(\dot\rho/\dot\sigma)\delta s\,. \tag{4.9}$$

While in the case with ${\rm d}\theta/{\rm d}\rho\neq0$ the evolution equations for ζ_n and s_n are coupled even at linear order, we can easily solve Eq. (4.7) up to $\mathcal{O}(\varepsilon)$. In the following calculations, we keep the terms up to this order, neglecting the terms of $\mathcal{O}(\varepsilon^2)$. Then, the constraint equations (4.5) and (4.6) are solved to give

$$\delta\tilde N\overset{\rm IR}{\approx}-\varepsilon_1\zeta_n\,,\qquad \frac1{\dot\rho}\partial_i\tilde N^i\overset{\rm IR}{\approx}\varepsilon_1\partial_\rho\zeta_n\,. \tag{4.10}$$

Using Eqs. (4.10), the equation of motion for ζ_n is recast into

$$\mathcal{L}\zeta_n\overset{\rm IR}{\approx}2\frac{{\rm d}\theta}{{\rm d}\rho}\partial_\rho s_n+\left(3\frac{{\rm d}\theta}{{\rm d}\rho}-\frac{V_{\sigma s}}{\dot\rho^2}\right)s_n-2\varepsilon_1\zeta_n\frac{e^{-2\rho}}{\dot\rho^2}\partial^2\zeta_n\,, \tag{4.11}$$

where the derivative operator $\mathcal{L}$ apparently takes the same form as the single field case:

$$\mathcal{L}:=\partial_\rho^2+(3-\varepsilon_1+\varepsilon_2)\partial_\rho-\frac{e^{-2\rho}}{\dot\rho^2}\partial^2\,. \tag{4.12}$$

Up to this order, interaction terms with the entropy field do not appear in the equation of motion for ζ_n. Allowing the introduction of homogeneous solutions, Eq. (4.11) is solved to give

$$\zeta_n\overset{\rm IR}{\approx}\psi+\varepsilon_1\psi\partial_\rho\psi+\xi_2\psi^2+\xi_3\psi^3$$
$$+\mu_2\psi\chi+\mu_3\psi^2\chi+\hat\mu_2\chi^2+\hat\mu_3\psi\chi^2\,, \tag{4.13}$$

where ξ_i, μ_i, and $\hat\mu_i$ for $i=2,3$ are time-dependent functions of $\mathcal{O}(\varepsilon)$. Noticing the fact that the interaction picture field of s_n satisfies

$$\mathcal{L}\chi=\mathcal{O}(\varepsilon)\,, \tag{4.14}$$

we also included the homogeneous solutions with χ.

B. IR regularity and gauge-invariance conditions

Now, we are ready to derive the gauge-invariance condition for the coupled case. Using Eqs. (2.18) and (2.19) together with Eqs. (4.8) and (4.13), we have

$${}^gR_2\overset{\rm IR}{\approx}\psi\partial^2\Big[\Big\{(1+\varepsilon_1)\partial_\rho-X^i\partial_{X^i}$$
$$+2\xi_2+\varepsilon_2/2\Big\}\psi+\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)\chi\Big]$$
$$+\chi\partial^2\Big[\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)\psi+2\hat\mu_2\chi\Big]\,, \tag{4.15}$$

and

$${}^gR_3\overset{\rm IR}{\approx}\psi^2\partial^2\Big[\frac12(\partial_\rho-X^i\partial_{X^i})^2\psi+3\xi_3\psi+\mu_3\chi$$
$$+(\partial_\rho-X^i\partial_{X^i})\Big\{(\varepsilon_1\partial_\rho+3\xi_2+3\varepsilon_2/4)\psi$$
$$+\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)\chi\Big\}\Big]$$
$$+2\psi\chi\partial^2\Big[\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)(\partial_\rho-X^i\partial_{X^i})\psi$$
$$+\hat\mu_2(\partial_\rho-X^i\partial_{X^i})\chi+\mu_3\psi+\hat\mu_3\chi\Big]. \tag{4.16}$$

We keep the terms in the last line of Eq. (4.15) and the terms in the last two lines of Eq. (4.16), which can yield the possibly divergent terms with $\langle\psi\chi\rangle$.

Making use of the Gram-Schmidt normalization, we can prepare a set of orthonormalized mode functions, which satisfy the Klein-Gordon normalization. We expand $\psi^I:=-(\dot\rho/\dot\sigma)\delta\phi^I$ by the orthonormalized mode function $\psi^I_{\alpha,\boldsymbol{k}}$ where $\alpha=1,2$ are the indices for the orthonormal bases. Using these orthonormal bases, ψ and χ are expanded as

$$\psi(X)=\sum_{\alpha=1}^2\Theta_{1I}\psi^I_\alpha(X)\,, \tag{4.17}$$

$$\chi(X)=\sum_{\alpha=1}^2\Theta_{2I}\psi^I_\alpha(X)\,, \tag{4.18}$$

where the time-dependent matrix Θ is already given in Eq. (4.1) and we defined $\psi^I_\alpha(X)$ as

$$\psi^I_\alpha(X)=\int\frac{{\rm d}^3\boldsymbol{k}}{(2\pi)^{3/2}}\left[\,\psi^I_{\alpha,\boldsymbol{k}}(X)a_{\alpha,\boldsymbol{k}}+{\rm h.c.}\right] \tag{4.19}$$

with the creation and annihilation operators that satisfy

$$\left[a_{\alpha,\boldsymbol{k}},\,a^\dagger_{\beta,\boldsymbol{k}'}\right]=\delta_{\alpha\beta}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,. \tag{4.20}$$

Substituting Eqs. (4.17) and (4.18) into Eqs. (4.15) and (4.16), the gauge-invariant spatial curvature is expanded as

$$\begin{aligned}{}^g\!R_2 \overset{\rm IR}{\approx} \psi\sum_{\alpha=1}^{2}\partial^2\Big[\Theta_{1I}\big\{(1+\varepsilon_1)\partial_\rho - X^i\partial_{X^i} \\ + 2\xi_2+\varepsilon_2/2\big\}+\Theta_{2I}\mu_2\Big]\psi^I_\alpha \\ +\chi\sum_{\alpha=1}^{2}\partial^2\Big[\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)\Theta_{1I}+2\hat\mu_2\Theta_{2I}\Big]\psi^I_\alpha,\end{aligned} \tag{4.21}$$

and

$$\begin{aligned}{}^g\!R_3 \overset{\rm IR}{\approx} \psi^2\sum_{\alpha=1}^{2}\partial^2\Big[\frac{1}{2}\Theta_{1I}(\partial_\rho - X^i\partial_{X^i})^2 \\ +\Theta_{1I}(3\xi_2+3\varepsilon_2/4+\varepsilon_1\partial_\rho)(\partial_\rho - X^i\partial_{X^i}) \\ +\Theta_{2I}\mu_2(\partial_\rho - X^i\partial_{X^i})+3\xi_3\Theta_{1I}+\mu_3\Theta_{2I}\Big]\psi^I_\alpha \\ +2\psi\chi\sum_{\alpha=1}^{2}\partial^2\Big[\Theta_{1I}\Big(\mu_2-\frac{{\rm d}\theta}{{\rm d}\rho}\Big)(\partial_\rho - X^i\partial_{X^i}) \\ +\Theta_{2I}\hat\mu_2(\partial_\rho - X^i\partial_{X^i})+\mu_3\Theta_{1I}+\hat\mu_3\Theta_{2I}\Big]\psi^I_\alpha\,,\end{aligned} \tag{4.22}$$

where we noted ${\rm d}\Theta_{1I}/{\rm d}\rho=({\rm d}\theta/{\rm d}\rho)\Theta_{2I}$.

In requesting the absence of the possibly divergent terms with $\langle\psi^2\rangle$ and $\langle\psi\chi\rangle$, we obtain the gauge-invariance conditions as

$$\begin{aligned}&\Big[\Theta_{1I}\big\{(1+\varepsilon_1)\partial_\rho - X^i\partial_{X^i}+2\xi_2+\varepsilon_2/2\big\}+\Theta_{2I}\mu_2\Big]\psi^I_{\alpha,\boldsymbol{k}} \\ &\qquad = -\Theta_{1I}D_k\psi^I_{\alpha,\boldsymbol{k}}\,, &(4.23)\\ &\xi_3=\mu_3=0\,, &(4.24)\end{aligned}$$

and

$$\mu_2=\frac{{\rm d}\theta}{{\rm d}\rho}\,,\qquad \hat\mu_2=\hat\mu_3=0\,. \tag{4.25}$$

If all these conditions are satisfied, the gauge-invariant spatial curvature ${}^gR$ is simply given by

$${}^g\!R_2 \overset{\rm IR}{\approx} -\psi\sum_{\alpha=1}^{2}\partial^2 D_k\Theta_{1I}\psi^I_\alpha, \tag{4.26}$$

$${}^g\!R_3 \overset{\rm IR}{\approx} \frac{1}{2}\psi^2\sum_{\alpha=1}^{2}\partial^2\Big[D_k^2\Theta_{1I}-\Big(2\xi_2+\frac{\varepsilon_2}{2}\Big)D_k\Theta_{1I}\Big]\psi^I_\alpha\,. \tag{4.27}$$

Then, the potentially divergent terms in ${}^gR$ are found to become the total derivative as

$$\begin{aligned}&\langle\{{}^g\!R(X_1),\,{}^g\!R(X_2)\}\rangle \\ &\overset{\rm IR}{\approx} \frac{1}{2}\langle\psi^2\rangle\Theta_{1I}\Theta_{1J}\int\frac{{\rm d}(\log k)}{2\pi^2}\Big\{\partial^2_{\log k}-\Big(2\xi_2+\frac{\varepsilon_2}{2}\Big)\partial_{\log k}\Big\} \\ &\qquad\times\sum_{\alpha=1}^{2}\big\{k^7\psi^I_{\alpha,\boldsymbol{k}}(X_1)\psi^{I\,*}_{\alpha,\boldsymbol{k}}(X_2)+({\rm c.c.})\big\}\,,\end{aligned} \tag{4.28}$$

and they vanish. We now understand that, after the choice of the appropriate initial condition, the two-point function of the gauge-invariant operator ${}^gR$ is shown to be regular, leaving a possibly IR divergent contribution with $\langle\chi^2\rangle$, which is irrelevant to the gauge effect.

We requested Eq. (4.25) to eliminate the possibly divergent terms with $\langle\psi\chi\rangle$. However, at first glance it may be unclear whether the realization of the gauge invariance truly requests the absence of the terms with $\langle\psi\chi\rangle$ or not, because the cross-correlation $\langle\psi\chi\rangle$ is also influenced by the entropy field. To make this point clear, we note that $\langle\psi\chi\rangle$ no longer diverges if at least one of ψ and χ is suppressed in the IR limit. If we remove the residual gauge degrees of freedom, say by adapting the local gauge condition [1], the adiabatic field is suppressed in the IR limit so that the regularity of $\langle\psi\chi\rangle$ is sufficiently guaranteed. This indicates that the cross-correlation $\langle\psi\chi\rangle$ does not diverge in the genuine gauge-invariant quantities. Therefore, we can request the absence of the IR divergence from $\langle\psi\chi\rangle$ as the necessary condition for the genuine gauge invariance.

Several remarks are in order regarding the gauge-invariance conditions (4.23) - (4.25). In harmony with the result in single field models, the gauge invariance condition (4.23) almost uniquely determine the mode function for ψ to that for the Bunch-Davies vacuum at the leading order in the slow-roll approximation. This can be confirmed by following a similar argument to the one in Refs. [2, 3]. The gauge-invariance conditions thus derived should be consistent with the equation of motion for the adiabatic field. Using the Fourier expanded basis:

$$\psi^I_{\alpha,\boldsymbol{k}}(X)=\frac{\dot\rho^2}{\dot\phi}\frac{1}{k^{3/2}}f^I_{\alpha,k}(\rho)\,e^{i\boldsymbol{k}\cdot\boldsymbol{X}}\,, \tag{4.29}$$

the gauge-invariance condition (4.23) can be recast into

$$\big\{(1+\varepsilon_1)\partial_\rho+\boldsymbol{k}\cdot\partial_{\boldsymbol{k}}+(2\xi_2-\varepsilon_1)\big\}\Theta_{1I}f^I_\alpha=0\,, \tag{4.30}$$

where we also used the first equation in Eq. (4.25). From Eq. (4.11), we have the mode equation at liner order as

$$\bar{\mathcal{L}}_k\Theta_{1I}f^I_{\alpha,k}-\Big(3\frac{{\rm d}\theta}{{\rm d}\rho}-\frac{V_{\sigma s}}{\dot\rho^2}+2\frac{{\rm d}\theta}{{\rm d}\rho}\partial_\rho\Big)\Theta_{2I}f^I_{\alpha,k}=0, \tag{4.31}$$

where we introduced the derivative operator $\bar{\mathcal{L}}_k$ as

$$\bar{\mathcal{L}}_k:=\partial_\rho^2+3(1-\varepsilon_1)\partial_\rho+\frac{e^{-2\rho}}{\dot\rho^2}k^2-3(\varepsilon_1+\varepsilon_2/2)\,. \tag{4.32}$$

Operating $\{(1+\varepsilon_1)\partial_\rho+\boldsymbol{k}\cdot\partial_{\boldsymbol{k}}\}$ on Eq. (4.31), which commutes with $\bar{\mathcal{L}}_k$ up to $\mathcal{O}(\varepsilon)$, we obtain

$$\begin{aligned}&\bar{\mathcal{L}}_k\{(1+\varepsilon_1)\partial_\rho+\boldsymbol{k}\cdot\partial_{\boldsymbol{k}}\}\Theta_{1I}f^I_{\alpha,k}\\&\quad-\left(3\frac{\mathrm{d}\theta}{\mathrm{d}\rho}-\frac{V_{\sigma s}}{\dot\rho^2}+2\frac{\mathrm{d}\theta}{\mathrm{d}\rho}\partial_\rho\right)(\partial_\rho+\boldsymbol{k}\cdot\partial_{\boldsymbol{k}})\Theta_{2I}f^I_{\alpha,k}=0\,.\end{aligned} \tag{4.33}$$

The mode function for the entropy field χ is not restricted from the gauge-invariance condition. However, to reproduce the gauge-invariance condition (4.30) from Eq. (4.33), we need to employ, at the leading order in the slow-roll approximation, the condition

$$(\partial_\rho-X^i\partial_{X^i}+D_k)\Theta_{2I}\psi^I_{\alpha,\boldsymbol{k}}=\mathcal{O}(\varepsilon)\,, \tag{4.34}$$

which almost determines the mode function for χ to that for the Bunch-Davies vacuum. Then, the terms in the second line of Eq. (4.33) vanish, reproducing Eq. (4.30) after the multiplication of $\bar{\mathcal{L}}_k^{-1}$. The last two terms in (4.30) appear as the homogeneous solutions of $\bar{\mathcal{L}}_k$. Following the same argument as in Ref. [3], we can also show that the gauge-invariance conditions and Eq. (4.34) sufficiently ensure that the mode equations are satisfied for all wavenumbers, if they are satisfied only a particular wavenumber.

C. Primordial non-Gaussianity

Now we calculate the bi-spectrum for ${}^g\zeta$, which again becomes the genuinely gauge-invariant variable for $1/L_{\rm obs}\ll k\ll e^\rho H$, in requesting the gauge-invariance conditions (4.23) - (4.25). Substituting Eqs. (2.16), (4.8), and (4.13) into Eq. (3.27), the gauge-invariant curvature perturbation ${}^g\zeta$ is given by

$${}^g\zeta\overset{\rm IR}{\approx}\psi+\psi(1+\varepsilon_1)\partial_\rho\psi-\psi X^i\partial_{X^i}\psi+\left(\frac14\varepsilon_2+\xi_2\right)\psi^2\,, \tag{4.35}$$

where we again abbreviated the several terms in δx^i which can give the non-vanishing contributions for $k\gg 1/L_{\rm obs}$. This is because we can show that these contributions independently vanish in the squeezed limit, repeating the same argument as in Ref. [4]. Note that after the use of Eq. (4.25), the terms with the entropy field disappear in the expression of ${}^g\zeta$. The expression of ${}^g\zeta$ includes the time-dependent function ξ_2, which is restricted from the request of the consistent quantization [3]. Assuming ξ_2 is determined appropriately, we do not discuss the explicit form of ξ_2, because this is not important for our discussions.

Using Eqs. (4.17) and (4.18), the gauge-invariant curvature perturbation ${}^g\zeta$ can be expanded as

$${}^g\zeta\overset{\rm IR}{\approx}\sum_{\alpha=1}^2\Theta_{1I}\psi^I_\alpha+\sum_{\alpha,\beta=1}^2\mathcal{M}_{\alpha\beta}\,, \tag{4.36}$$

where $\mathcal{M}_{\alpha\beta}$ is defined as

$$\mathcal{M}_{\alpha\beta}:=\Theta_{1I}\psi^I_\alpha\left[(1+\varepsilon_1)\partial_\rho-X^i\partial_{X^i}+\left(\frac14\varepsilon_2+\xi_2\right)\right]\Theta_{1J}\psi^J_\beta\,. \tag{4.37}$$

The bispectrum for ${}^g\zeta$ is again given by the three terms in Eq. (3.29), where only the first two terms give dominant contributions in the squeezed limit $k_1\ll k_2\simeq k_3$. Using Eq. (4.36), the first term in Eq. (3.29) is given by

$$\begin{aligned}&\langle\psi_{\boldsymbol{k}_1}\psi_{\boldsymbol{k}_2}{}^g\zeta_{\boldsymbol{k}_3,2}\rangle\\&\quad\overset{\rm IR}{\approx}\Theta_{1I}\Theta_{1J}\sum_{\alpha,\beta,\alpha',\beta'=1}^2\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\beta,\boldsymbol{k}_2}\mathcal{M}_{\alpha'\beta',\boldsymbol{k}_3}\rangle\,,\end{aligned} \tag{4.38}$$

where $\mathcal{M}_{\alpha\beta,\boldsymbol{k}}$ is the Fourier mode of $\mathcal{M}_{\alpha\beta}$. Note that $\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\beta,\boldsymbol{k}_2}\mathcal{M}_{\alpha'\beta',\boldsymbol{k}_3}\rangle$ gives the non-vanishing contributions only if α and β agree with either α' or β', while α and β do not necessarily coincide with each other. We first consider the case with $\alpha=\beta$, where the non-vanishing contribution is given by

$$\begin{aligned}&\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\alpha,\boldsymbol{k}_2}\mathcal{M}_{\alpha\alpha,\boldsymbol{k}_3}\rangle\\&\overset{\rm IR}{\approx}-v^I_{\alpha,k_1}v^J_{\alpha,k_2}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3\boldsymbol{X}}{(2\pi)^3}\\&\quad\times\Big[e^{-i(\boldsymbol{k}_1+\boldsymbol{k}_3)\cdot\boldsymbol{X}}\delta^{(3)}(\boldsymbol{k}_2+\boldsymbol{p})\psi^*_{\alpha,k_1}D_p\psi^*_{\alpha,p}e^{i\boldsymbol{p}\cdot\boldsymbol{X}}\\&\qquad+e^{-i(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{X}}\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{p})\psi^*_{\alpha,k_2}X^i\partial_{X^i}\psi^*_{\alpha,p}e^{i\boldsymbol{p}\cdot\boldsymbol{X}}\Big]\\&\quad+(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\,v^I_{\alpha,k_1}v^J_{\alpha,k_2}\psi^*_{\alpha,k_2}\partial_\rho\psi^*_{\alpha,\,k_1}\,,\end{aligned} \tag{4.39}$$

where $\psi_{\alpha,k}$ is defined as $\psi_{\alpha,k}:=\Theta_{1I}v^I_{\alpha,k}$ and $\boldsymbol{K}$ is defined as $\boldsymbol{K}:=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3$. In deriving this expression, we used the gauge-invariance condition (4.23) and $\partial_\rho v^I_\alpha=\mathcal{O}(\varepsilon)$ for $k\ll e^\rho H$. Next, for the case with $\alpha\neq\beta$, we obtain

$$\begin{aligned}&\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\beta,\boldsymbol{k}_2}\left(\mathcal{M}_{\alpha\beta,\boldsymbol{k}_3}+\mathcal{M}_{\beta\alpha,\,\boldsymbol{k}_3}\right)\rangle\\&\overset{\rm IR}{\approx}-v^I_{\alpha,k_1}v^J_{\beta,k_2}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3\boldsymbol{X}}{(2\pi)^3}\\&\quad\times\Big[e^{-i(\boldsymbol{k}_1+\boldsymbol{k}_3)\cdot\boldsymbol{X}}\delta^{(3)}(\boldsymbol{k}_2+\boldsymbol{p})\psi^*_{\alpha,k_1}D_p\psi^*_{\beta,p}e^{i\boldsymbol{p}\cdot\boldsymbol{X}}\\&\qquad+e^{-i(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{X}}\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{p})\psi^*_{\beta,k_2}X^i\partial_{X^i}\psi^*_{\alpha,p}e^{i\boldsymbol{p}\cdot\boldsymbol{X}}\Big]\\&\quad+(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\,v^I_{\alpha,k_1}v^J_{\beta,k_2}\psi^*_{\beta,k_2}\partial_\rho\psi^*_{\alpha,k_1}\,.\end{aligned} \tag{4.40}$$

The calculation of the second term in Eq. (3.29) proceeds similarly to give

$$\begin{aligned}&\langle\psi_{\boldsymbol{k}_1}{}^g\zeta_{\boldsymbol{k}_2,2}\psi_{\boldsymbol{k}_3}\rangle\\&\quad\approx\Theta_{1I}\Theta_{1J}\sum_{\alpha,\beta,\alpha',\beta'=1}^2\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\mathcal{M}_{\alpha'\beta',\boldsymbol{k}_2}\psi^J_{\beta,\boldsymbol{k}_3}\rangle\,,\end{aligned} \tag{4.41}$$

where the non-vanishing contributions are related to those in

the first term as

$$\begin{aligned}
&\Theta_{1I}\Theta_{1J}\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\mathcal{M}_{\alpha\alpha,\boldsymbol{k}_2}\psi^J_{\alpha,\boldsymbol{k}_3}\rangle \\
&\quad-(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\;|\psi_{\alpha,k_3}|^2\,\psi_{\alpha,k_1}\partial_\rho\psi^*_{\alpha,k_1} \\
&\overset{\text{IR}}{\approx}\Big[\Theta_{1I}\Theta_{1J}\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\alpha,\boldsymbol{k}_3}\mathcal{M}_{\alpha\alpha,\boldsymbol{k}_2}\rangle \\
&\quad-(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\,|\psi_{\alpha,k_3}|^2\,\psi_{\alpha,k_1}\partial_\rho\psi^*_{\alpha,k_1}\Big]^*,
\end{aligned} \tag{4.42}$$

and for $\alpha\neq\beta$

$$\begin{aligned}
&\Theta_{1I}\Theta_{1J}\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\left(\mathcal{M}_{\alpha\beta,\boldsymbol{k}_2}+\mathcal{M}_{\beta\alpha,\,\boldsymbol{k}_2}\right)\psi^J_{\beta,\boldsymbol{k}_3}\rangle \\
&\quad-(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\;|\psi_{\beta,k_3}|^2\,\psi_{\alpha,k_1}\partial_\rho\psi^*_{\alpha,k_1} \\
&\overset{\text{IR}}{\approx}\Big[\Theta_{1I}\Theta_{1J}\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\beta,\boldsymbol{k}_3}\left(\mathcal{M}_{\alpha\beta,\boldsymbol{k}_2}+\mathcal{M}_{\beta\alpha,\,\boldsymbol{k}_2}\right)\rangle \\
&\quad-(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\,|\psi_{\beta,k_3}|^2\,\psi_{\alpha,k_1}\partial_\rho\psi^*_{\alpha,k_1}\Big]^*.
\end{aligned} \tag{4.43}$$

Now, we consider the squeezed limit $k_1\ll k_2\simeq k_3$. After some manipulation, Eq. (4.39) multiplied by $\Theta_{1I}\Theta_{1J}$ can be expressed as

$$\begin{aligned}
&\Theta_{1I}\Theta_{1J}\langle\psi^I_{\alpha,\,\boldsymbol{k}_1}\psi^J_{\alpha,\boldsymbol{k}_2}\mathcal{M}_{\alpha\alpha,\boldsymbol{k}_3}\rangle \\
&\overset{\text{IR}}{\approx}-\frac{1}{2}\,|\psi_{\alpha,k_1}|^2\,\psi_{\alpha,k_2}\psi^*_{\alpha,|\boldsymbol{k}_1+\boldsymbol{k}_3|} \\
&\qquad\qquad\times(2\pi)^{-3/2}(\boldsymbol{k}_1-\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\partial_{\boldsymbol{K}}\delta^{(3)}(\boldsymbol{K}) \\
&\quad-|\psi_{\alpha,k_1}|^2\,|\psi_{\alpha,k_2}|^2\,(2\pi)^{-3/2}\boldsymbol{k}_1\cdot\partial_{\boldsymbol{K}}\delta^{(3)}(\boldsymbol{K}) \\
&\quad+(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\,\psi_{\alpha,k_1}\,|\psi_{\alpha,k_2}|^2\,\partial_\rho\psi^*_{\alpha,\,k_1}.
\end{aligned} \tag{4.44}$$

(The detailed calculation can be consulted in Ref. [4].) The terms with $\boldsymbol{k}_1$ manifestly vanish in the squeezed limit $k_1\to 0$. The rest terms on the first and second lines are not suppressed at this moment. Combining the contribution from the second term of Eq. (3.29), which satisfies Eq. (4.42), these terms, however, provide the factor $(\psi_{\alpha,k_2}\psi^*_{\alpha,|\boldsymbol{k}_1+\boldsymbol{k}_3|}-\psi_{\alpha,|\boldsymbol{k}_1+\boldsymbol{k}_2|}\psi^*_{\alpha,k_3})$, which again vanishes in the squeezed limit, $k_1\to 0$. We therefore understand that only the term on the last line in Eq. (4.44) yields the non-vanishing contribution in this limit. Repeating a similar calculation, we again find that all the terms in Eq. (4.40) except for the last one vanish in this limit. Finally, the bispectrum for ${}^g\zeta$ in the squeezed limit $k_1\to 0$ is given by

$$\begin{aligned}
&\langle{}^g\zeta_{\boldsymbol{k}_1}{}^g\zeta_{\boldsymbol{k}_2}{}^g\zeta_{\boldsymbol{k}_3}\rangle \\
&\approx 2(2\pi)^{-3/2}\delta^{(3)}(\boldsymbol{K})\sum_{\alpha,\beta=1}^{2}|\psi_{\alpha,k_2}|^2\,\mathrm{Re}\big[\psi_{\beta,k_1}\partial_\rho\psi^*_{\beta,k_1}\big]\ ,
\end{aligned} \tag{4.45}$$

Note that for $\mathrm{d}\theta/\mathrm{d}\rho=0$ the right-hand side of Eq. (4.45) vanishes, reproducing the result in Eq. (3.31), because for this particular case $\psi_{\alpha,k}$ becomes constant in time at superhorizon scales. By contrast, for $\mathrm{d}\theta/\mathrm{d}\rho\neq 0$ the adiabatic field does not necessarily become constant. The result in Eq. (4.45) indicates that the time variation in the local rotation angle can generate the observable fluctuation, which are not eliminated by gauge transformations.

V. CONCLUSION

In this paper we studied the implications of the genuine gauge invariance in two-field models of inflation. We showed that, likewise in single field models, if initial quantum states satisfy the gauge-invariance conditions, the loop diagrams with the adiabatic field do not yield IR divergences. It is remarkable that the gauge-invariance conditions, imposed on the adiabatic field, can be influenced by the entropy field. This is because the interactions between the adiabatic and entropy fields can generate the additional possibly divergent terms, which are absent in single field models, and these contributions are also influenced by gauge effects. For the derivation of the gauge-invariance conditions, we distinguished the IR divergences which are relevant to gauge effects from those which are irrelevant to gauge effects, considering the diagrams up to one-loop order. This discrimination would become rather complicated if we extend our argument to higher order in loops. The loop correction of the entropy field $\langle\delta s\,\delta s\rangle$ then comes to be no longer gauge invariant due to the contamination of the adiabatic field.

In requesting the gauge invariance in the local universe, we reexamined the bispectrum for the primordial curvature perturbation. The conventionally used curvature perturbation ζ preserves the invariance under normalizable gauge transformations, but it does not under non-normalizable gauge transformations. This indicates that the curvature perturbation ζ does not preserve the gauge-invariance in the local universe while this should be preserved in observable fluctuations. We therefore calculated the tree-level bi-spectrum for the genuine gauge-invariant curvature ${}^g\zeta$ to discuss observable fluctuations. In contrast to the result in single field models, where the genuine gauge-invariant bi-spectrum completely vanishes in the squeezed limit, in multi-field models, we still have the non-vanishing contributions in this limit. This is generated from the time variation in the curvature perturbation at superhorizon scales, which occurs only if the background trajectory is curved yielding $\mathrm{d}\theta/\mathrm{d}\rho\neq 0$. This effect can be understood as follows. While the constant part of the curvature perturbation can be removed by a local dilatation, which is the non-normalizable gauge transformation, the time dependent part cannot be removed by gauge transformations and produce the physical effect. We should emphasize that our arguments can be applied to the fluctuations in the observable scales of current measurements. Since the gauge-invariance conditions derived here are the necessity conditions, precisely speaking, the genuine gauge invariance of the bi-spectrum for ${}^g\zeta$ has not sufficiently proven. It is, however, intuitively reasonable to expect the non-vanishing physical effects in the presence of the entropy field. In this paper, we employed the slow-roll approximation. It would be interesting to discuss the observable effects in models which break the slow-roll approximation, where large non-Gaussianities are predicted in the conventional perturbation theory.

At the end of this paper, we add several comments on the IR divergence from the entropy field. The loop corrections of the entropy field can be divergent if the entropy field has the scale-invariant or red-tilted spectrum. This divergence cannot be

eliminated also in genuine gauge-invariant quantities. The IR divergence from the entropy field should be regularized by a different way from the one for the adiabatic field. In Ref. [36], we proposed one way to regularize the IR divergence from the entropy fields. (Some other ways of regularization were discussed in Refs. [44–48].) We showed that, if we take into account the effects of the quantum decoherence which pick up a unique history of the universe from various possibilities contained in initial quantum state, the IR loop corrections of the entropy field no longer diverge. Therefore, if we consider the decoherence effect, it would be possible to show the IR regularity of the genuine gauge-invariant quantities.

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